

## Comparing Mutually Exclusive Projects with Unequal Lives

Chapters 11 and 12 covered the basic principles of capital budgeting. In this web appendix, we consider mutually exclusive projects with unequal lives. Up until now, we have treated projects as “one-shot” occurrences. However, some projects can be repeated over time. Therefore, when choosing between mutually exclusive projects, we must first determine whether they can be repeated; and if so, we must consider this fact when we estimate the projects’ profitability.

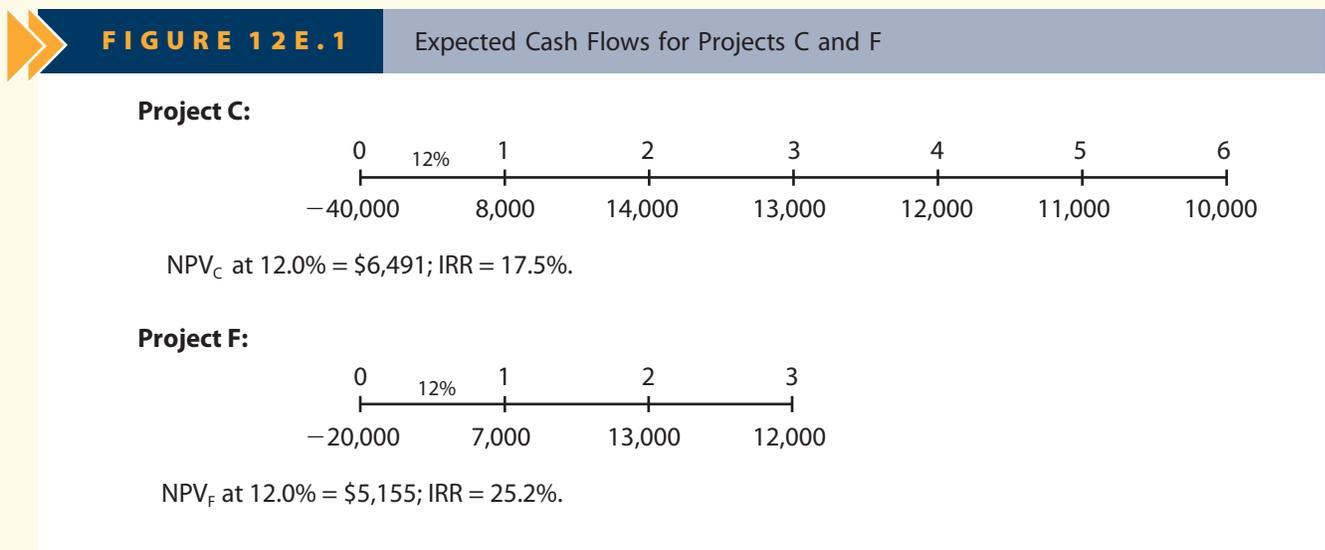
If a company is choosing between two mutually exclusive alternatives with significantly different lives, an adjustment may be necessary. For example, suppose a firm is planning to modernize its distribution center. It is considering either a conveyor system (Project C) or some forklift trucks (Project F) for moving goods. Figure 12E.1 shows the expected cash flows and the NPVs for these two mutually exclusive alternatives. We see that Project C, when discounted at a 12% WACC, has the higher NPV and thus appears to be the better project.

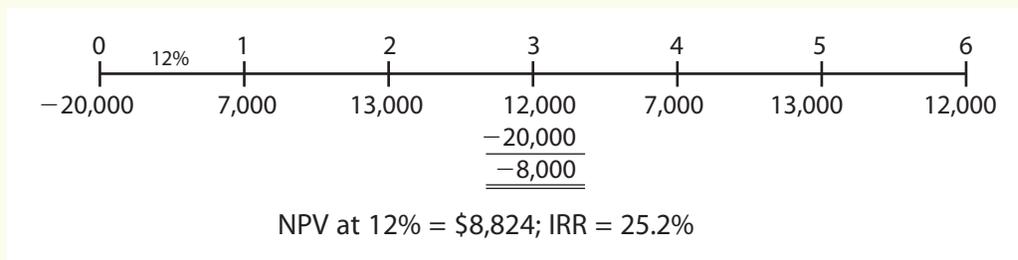
Although the analysis in Figure 12E.1 suggests that Project C should be selected, this analysis is incomplete and the decision to choose Project C is incorrect. If we choose Project F, we will have an opportunity to make a similar investment in three years; and if cost and revenue conditions remain at the Figure 12E.1 levels, this second investment will also be profitable. However, if we choose Project C, we will not have this second investment opportunity. Therefore, to make a proper comparison between Projects C and F, we would apply the **replacement chain (common life) approach**. This involves finding the NPV of Project F over a six-year period, and then comparing this extended NPV with the NPV of Project C over the same 6 years.

The NPV for Project C as calculated in Figure 12E.1 is already over the 6-year common life. For Project F, however, we must add a second project to extend the overall life of the combined projects to 6 years. Here we assume (1) that Project F’s costs and annual cash inflows will not change if the project is repeated in 3 years and (2) that the firm’s WACC will remain at 12%:

### Replacement Chain (Common Life) Approach

*A method for comparing projects of unequal lives that assumes that each project can be repeated as many times as necessary to reach a common life span; the NPVs over this life span are then compared, and the project with the higher common-life NPV is chosen.*





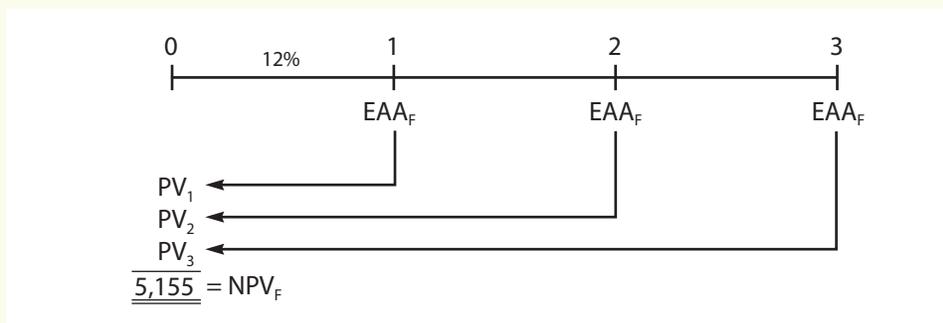
The NPV of this extended Project F is \$8,824, and its IRR is 25.2%. (The IRR of two Project Fs is the same as the IRR for one Project F.) Since F's \$8,824 extended NPV is greater than the \$6,491 NPV of Project C, Project F should be selected.

### EQUIVALENT ANNUAL ANNUITY (EAA) APPROACH

Although the preceding example illustrates why an extended analysis is necessary if we are comparing mutually exclusive projects with different lives, the arithmetic is generally more complex in practice. For example, one project might have a 6-year life versus a 10-year life for the other project. This would require a replacement chain analysis over 30 years, the lowest common denominator of the two lives. In such a situation, it is often simpler to use a second procedure, the **equivalent annual annuity (EAA) method**, which involves three steps:

**Equivalent Annual Annuity (EAA) Method**  
 A method for comparing projects of unequal lives that calculates the annual payments a project would provide if it were an annuity. When comparing projects of unequal lives, the one with the higher equivalent annual annuity should be chosen.

1. Find each project's NPV over its initial life. In Figure 12E.1, we found  $NPV_C = \$6,491$  and  $NPV_F = \$5,155$ .
2. There is some constant annuity cash flow (the equivalent annual annuity [EAA]) that has the same present value as a project's calculated NPV. For Project F, here is the time line:



To find the value of  $EAA_F$  with a financial calculator, enter  $N = 3$ ,  $r = I/YR = 12$ ,  $PV = -5155$ , and  $FV = 0$ , and solve for  $PMT$ . The answer is \$2,146.28. This level cash flow stream, when discounted back 3 years at 12%, has a present value equal to Project F's original NPV, \$5,155. The \$2,146.28 is called the project's equivalent annual annuity (EAA). The EAA for Project C can be found similarly, and it is \$1,578.78. Thus, Project C has an NPV which is equivalent to an annuity of \$1,578.78 per year, while Project F's NPV is equivalent to an annuity of \$2,146.28.<sup>1</sup>

<sup>1</sup>Some financial calculators are programmed with the EAA feature. For example, the HP-17BII has the function in its cash flow register. One simply keys in the cash flows, enters the interest rate, and then presses the "NUS" key to calculate the EAA. Hewlett-Packard uses the term NUS (for "net uniform series") in lieu of the term EAA.

3. The project with the higher EAA will always have the higher NPV when extended out to any common life. Therefore, because F's EAA is larger than C's, we would choose Project F.

The EAA method is often easier to apply than the replacement chain method, but the replacement chain method is easier to explain to decision makers. Still, the two methods lead to the same decision if consistent assumptions are used.

When should we worry about unequal life analysis? As a general rule, the unequal life issue (1) does not arise for independent projects but (2) can arise if mutually exclusive projects with significantly different lives are being compared. However, even for mutually exclusive projects, it is not always appropriate to extend the analysis to a common life. This should be done only when there is a high probability that the projects will be repeated at the end of their initial lives.

We should note several potentially serious weaknesses inherent in this type of analysis: (1) If inflation is expected, replacement equipment will have a higher price. Moreover, both sales prices and operating costs will probably change. Thus, the static conditions built into the analysis would be invalid. (2) Replacements that occur later will probably employ new technology, which in turn might change the cash flows. This factor can be built into the replacement chain analysis but not into the EAA approach. (3) It is difficult enough to estimate the lives of most projects, so estimating the lives of a series of projects is often just speculation.

In view of these problems, no experienced financial analyst would be too concerned about comparing mutually exclusive projects with lives of, say, 8 years and 10 years. Given all the uncertainties in the estimation process, analysts would assume that, for all practical purposes, such projects would have the same life. Still, it is important to recognize that a problem exists when mutually exclusive projects have substantially different lives. When we encounter such problems in practice, we use a computer spreadsheet and build expected inflation and/or possible efficiency gains directly into the cash flow estimates, and then use the replacement chain approach (but not the equivalent annual annuity method).

## QUESTIONS

- 12E-1** Why are the replacement chain and equivalent annuity methods important in the evaluation of mutually exclusive projects with different operating lives?
- 12E-2** Which method is better or easier: the replacement chain or equivalent annual annuity method? Explain.
- 12E-3** What are some potential weaknesses of analyses using the replacement chain or equivalent annual annuity methods?

## PROBLEMS

- 12E-1** **UNEQUAL LIVES** Haley's Graphic Designs Inc. is considering two alternative, mutually exclusive projects. Both projects require an initial investment of \$10,000 and are typical, average-risk projects for the firm. Project A has an expected life of 2 years with after-tax cash inflows of \$6,000 and \$8,000 at the end of Years 1 and 2, respectively. Project B has an expected life of 4 years with after-tax cash inflows of \$4,000 at the end of each of the next 4 years. The firm's WACC is 10%.
- If the projects cannot be repeated, which project should be selected if Haley uses NPV as its criterion for project selection?
  - Assume that the projects can be repeated and that there are no anticipated changes in the cash flows. Use the replacement chain analysis to determine the NPV of the project selected.
  - Make the same assumptions as in Part b. Use the equivalent annual annuity method to determine the annuity of the project selected.

- 12E-2 UNEQUAL LIVES** Cotner Clothes Inc. is considering the replacement of its old, fully depreciated knitting machine. Two new models are available: (a) Machine 190-3, which has a cost of \$190,000, a 3-year expected life, and after-tax cash flows (labor savings and depreciation) of \$87,000 per year, and (b) Machine 360-6, which has a cost of \$360,000, a 6-year life, and after-tax cash flows of \$98,300 per year. Assume both projects can be repeated. Knitting machine prices are not expected to rise because inflation will be offset by cheaper components (microprocessors) used in the machines. Assume that Cotner's WACC is 14%. Use both the replacement chain and EAA approaches to arrive at your answer.
- 12E-3 REPLACEMENT CHAIN** Zappe Airlines is considering two alternative planes. Plane A has an expected life of 5 years, will cost \$100 million, and will produce cash flows of \$30 million per year. Plane B has a life of 10 years, will cost \$132 million, and will produce cash flows of \$25 million per year. Zappe plans to serve the route for 10 years. The company's WACC is 12%. If Zappe needs to purchase a new Plane A, the cost will be \$105 million but cash inflows will remain the same. Should Zappe acquire Plane A or Plane B? Explain your answer.
- 12E-4 REPLACEMENT CHAIN** The Fernandez Company has the opportunity to invest in one of two mutually exclusive machines that will produce a product that the company will need for the next 8 years. Machine A costs \$10 million but will provide after-tax inflows of \$4 million per year for 4 years. If Machine A is replaced, its cost will be \$12 million due to inflation and its cash inflows will increase to \$4.2 million due to production efficiencies. Machine B costs \$15 million and will provide after-tax inflows of \$3.5 million per year for 8 years. If the WACC is 10%, which machine should Fernandez acquire?